

## Causality of the Brane Universe - OPERA and ICARUS

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### Abstract

The apparent violation of causality in the brane Universe can be avoided by taking the bulk spacetime modelled by a 5-dimensional Kaluza theory with factorizable ansatz for the 5-dimensional metric whose components do not depend on the fifth coordinate and with  $G_{55}$  not a constant. The geodesic in the bulk does not correspond to a free particle. The Kaluza scalar makes it non-inertial. The implications on the neutrino experiment is that there is no superluminal propagation even after invoking sterile neutrinos.

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The idea that our Universe is a brane in some higher dimensional spacetime with the matter (physical) fields are confined in the brane and the gravitational fields live in the brane and higher dimensional spacetime, is gaining importance since the work of Randall and Sundrum [1]. In Ref.1, the brane is embedded in a 5-dimensional spacetime (with warped metric) without compactifying the fifth dimension. A possibility that matter in the brane may be connected almost instantaneously through the fifth dimension has been suggested by Kälbermann and Halevi [2]. The causal structure of the brane Universe with possible apparent violation of causality has been demonstrated by Ishihara [3]. This exotic scenario gained prominence recently to understand the results of the OPERA experiment on muon-neutrino beam [4] and many attempts were made to reconcile with the OPERA data [5,6,7,8] using the brane world approach. In these studies, the existence of 'sterile neutrinos' is assumed. The 'active neutrinos' (in the Standard Model) propagate in the (3+1)-dimensional brane while the 'sterile neutrinos' (gauge singlets) are free to propagate in the extra-dimensional bulk as well as in the brane. Its geodesic between two points on the brane will include travel in bulk spacetime. By introducing a mixing of the sterile and active neutrinos, the studies show that the active neutrinos appear to be superluminal. There are other studies [9,10,11,12,13,14,15,16,17,18, partial list] on OPERA result not invoking brane world scenario and even suspecting the experiment [19].

Whether the results of the OPERA experiment are correct or not, the idea that there is a possible apparent violation of causality for the brane Universe merits further study. This becomes relevant in view of the recent experiment (ICARUS Collaboration) [20] that the time of flight difference between the speed of light and the arriving neutrino events is compatible with the simultaneous arrival of all events with equal speed.

Briefly, Ishihara [3] considered the brane as a 4-dimensional intrinsically flat spacetime. Since the brane is embedded in the higher dimensional bulk spacetime, taken as 5-dimensional, there is the induced metric on the brane and an extrinsic curvature. The brane is curved extrinsically by its self gravity. It is then possible that a path appears in the bulk corresponding to two points on the brane. There will be two paths: a path confined on the brane with induced metric and extrinsic curvature and a path in the bulk which is free. If the information through the path in the bulk arrives earlier than the one

through the path in the brane, then there is an apparent violation of causality. Ishihara [3] indeed showed that this happens when the brane is concave towards the bulk in the null direction. In other words, the fluctuations in the brane (via the extrinsic curvature) effectively increase the path length in the brane relative to the path length in the bulk. The 'on-brane geodesic' is longer than the 'bulk geodesic'. This conclusion is based on the realization that the geodesic in the bulk corresponds to a free particle.

In this note, it will be shown that the geodesic in the bulk does not correspond to a free particle. As an illustrative model for the bulk, we consider it a 5-dimensional spacetime as in the case of Kaluza-Klein theory. We assume a factorizable ansatz for the metric in the bulk for simplicity. The fifth coordinate is not taken to be compact. Further, the electromagnetism is switched off as photons live in the brane only. The 5-dimensional metric in the bulk is taken to be

$$G_{AB} = \begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & G_{55} \end{pmatrix} ; \quad G^{AB} = \begin{pmatrix} g^{\mu\nu} & 0 \\ 0 & \frac{1}{G_{55}} \end{pmatrix}, \quad (1)$$

where  $A = \{\mu, 5\}$ . We do not consider  $G_{55}$  a constant. All the entries in the metric are taken to be independent of the fifth coordinate but depend on  $x^\mu$ . The above metric is taken as a model for the bulk. The brane world is the physical 4-dimensional spacetime with metric  $g_{\mu\nu}(x)$ . It is to be recalled that in the Kaluza theory which unifies gravity with electromagnetism, the  $G_{55}$  component should not be a constant, though many authors considered this. With  $G_{55}$  a constant, the Einstein equations in the 5-dimensional world become inconsistent. This important result was shown by Jordon [21] and Thiry [22] and reviewed by Overduin and Wesson [23]. In our case, with  $G_{55}$  not a constant, the Einstein equations in the bulk for  $\tilde{R}_{55}, \tilde{R}_{\mu 5}, \tilde{R}_{\mu\nu}$  are consistent, in the absence of electromagnetism.

The geodesic equation in the 5-dimensional bulk spacetime is

$$\frac{d^2 Z^A}{ds^2} + \Delta_{BC}^A \frac{dZ^B}{ds} \frac{dZ^C}{ds} = 0, \quad (2)$$

where  $Z^A = \{x^\mu, x^5\}$  and  $\Delta_{BC}^A$ 's are the 5-dimensional Christoffel connections;  $\Delta_{BC}^A = \frac{1}{2} G^{AD} (\partial_B G_{CD} + \partial_C G_{BD} - \partial_D G_{BC})$  with  $\partial_A = \frac{\partial}{\partial Z^A}$ . We rewrite

(2) as

$$\frac{d}{ds} \left( G_{AB} \frac{dZ^B}{ds} \right) - \frac{1}{2} (\partial_A G_{CD}) \frac{dZ^C}{ds} \frac{dZ^D}{ds} = 0. \quad (3)$$

The geodesics (2) or (3) are the geodesics for a free particle in the 5-dimensional bulk. Since the metric  $G_{AB}$  in (1) are taken to be independent of the fifth coordinate, the  $A = 5$  part of (3) gives  $G_{5B} \frac{dZ^B}{ds}$  a constant along the geodesic. We shall denote this constant by  $a$ . The  $A = \mu$  part of (3) then becomes

$$\frac{d}{ds} \left( g_{\mu\nu} \frac{dx^\nu}{ds} \right) - \frac{1}{2} (\partial_\mu g_{\nu\rho}) \frac{dx^\nu}{ds} \frac{dx^\rho}{ds} = \frac{1}{2} \frac{a^2}{G_{55}^2} (\partial_\mu G_{55}), \quad (4)$$

which can be expressed as

$$\frac{d^2 x^\mu}{ds^2} + \Gamma_{\rho\nu}^\mu \frac{dx^\rho}{ds} \frac{dx^\nu}{ds} = \frac{1}{2} \frac{a^2}{G_{55}^2} g^{\mu\lambda} (\partial_\lambda G_{55}), \quad (5)$$

where  $\Gamma_{\rho\nu}^\mu$  is the Christoffel connection for the metric  $g_{\mu\nu}(x)$ . This is the geodesic equation for a particle in the bulk 5-dimensional spacetime with indices  $\mu, \nu, \rho, \lambda$  taking values  $(0, 1, 2, 3)$  as in the brane world. This describes the path appearing in the bulk corresponding to two points on the brane. Thus the path in the bulk, corresponding to two points on the brane, is not a 'free path'. The Kaluza scalar (the  $G_{55}$  part of  $G_{AB}$  in (1)) provides an additional force on the particle in the bulk. In this model, the 'on-brane geodesic' will not be longer than the 'bulk geodesic' as the geodesic in the bulk does not correspond to a free particle while the 'on-brane geodesic' corresponds to a free particle.

If now, the fluctuations in the brane are taken into account by means of extrinsic curvature, then the geodesic on the brane has a contribution from the extrinsic curvature. When the brane is concave towards the bulk in the null direction, it is conceivable that the roles of the extrinsic curvature effects on the geodesic in the brane and that of the Kaluza scalar  $G_{55}(x)$  on the geodesic in the bulk nicely balance so that the particles following the two paths, one on the brane and the other on the bulk, arrive eventually at the same time on the brane. This avoids the violation of causality in the brane Universe.

Now analysing the reasoning of the explanation of the OPERA result by invoking sterile and active neutrinos with mixing, we see that the sterile neutrinos travelling in the bulk and the active neutrinos travelling in the brane could arrive at the same time on the brane, thereby negating the superluminal propagation of neutrinos. It is gratifying that this is in agreement with the results of the recent ICARUS Collaboration [20] that the time of flight difference between the speed of light and the arriving neutrino events is compatible with the simultaneous arrival of all the events.

The Kaluza's five dimensional spacetime description of the bulk (in which the Standard Model matter fields do not exist) - a model - with  $G_{55}$  not a constant, is further examined in the light of constraints by Gubser [24]. Gubser [24] considered an extra-dimensional space with a non-factorizable ansatz for the metric and showed a possible violation of the null energy condition in extra dimensions. In our case, we have a factorizable metric in (1) and none of the components of  $G_{AB}$  depend on the extra coordinate. Further, the Einstein equations in the bulk are  $\tilde{R}_{AB} - \frac{1}{2}G_{AB}\tilde{R} = 0$  and so the null energy condition  $\tilde{R}_{AB}\xi^A\xi^B = 0$  for  $\xi^A$  any null vector is satisfied. So the constraint does not arise.

To summarize: The apparent violation of causality in the brane Universe can be avoided by taking the bulk spacetime modelled by a 5-dimensional Kaluza theory with factorizable ansatz for the 5-dimensional metric whose components do not depend on the fifth coordinate and with  $G_{55}$  not a constant. The geodesic in the bulk does not correspond to a free particle. The Kaluza scalar makes it non-inertial. The geodesic in the brane having extrinsic curvature contribution makes it possible that the lengths of the geodesics in the bulk and on the brane can be made same, thereby avoiding the violation of causality in the brane Universe. The implications on the neutrino experiment is that there is no superluminal propagation even invoking sterile neutrinos. We have considered the fifth coordinate as non-compact. The conclusions remain unaltered if the fifth coordinate is compactified to a circle. In this case, we need to retain the  $n = 0$  mode only as this corresponds to the lowest (vacuum) state.

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